RATE CONTROL SYSTEM FOR PLANT PARAMETERS UNCERTAINTIES

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ABSTRACT

A Rate Control System is used in Sounding Rocket Payload for microgravity experiments to reduce its angular velocity to get improvement at microgravity environment during the payload ballistic phase without atmosphere influence. The design of the rate control system may be developed based on plant parameters, that is, the payload mathematical model. But parameters uncertainties of the real payload dynamic can compromise the performance of the rate control system. Using the state plane method to design, the Rate Control System presents a good performance in presence of mass, actuator torque level or transient parameters uncertainties and noise at sensors measure. It'll be shown the state plane method to design the Rate Control System and its performance to the parameters uncertainties mentioned above.

1. INTRODUCTION

Some class of microgravity experiments uses sounding rocket that carries a Payload with experiments outside the Earth atmosphere influence to perform microgravity experiments during its ballistic phase. After sounding rocket separation the payload must have its angular velocity reduced as close as possible to zero and keep this way during this ballistic phase. To get and keep angular velocity near to zero the payload needs a Rate Control System (RCS). There are several methods to design a RCS and many of them are mathematical base-model of the payload and the actuator. To reduce the costs and complexity of the RCS it is used actuators based on Cold Gas System. The basic problem for this kind of control is to determine the pulse width to activation and deactivated the actuators to reduce and keep the payload angular velocity as close as possible to zero. But there are parameters uncertainties between the payload and the mathematical model that can compromise the RCS performance, such as: payload mass and inertia variation, actuator torque level and direction error, sensors error measure, noise, motor separation, structural vibration, etc.

At this work it'll be shown the state plane method to design a RCS and its performance due the plant mass and actuator transient response uncertainties and sensors measurement corrupted by noise.

A RCS can be comprised basically by the units showed at Fig. 1:

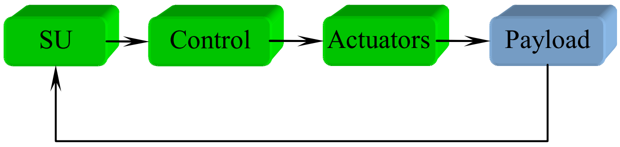

Figure 1 – RCS and Payload Diagram

where SU is the sensor unit that measures the angular movement of the payload at three main payload axes. The control unit has a Central Processor Unit (CPU) that is programmed with the RCS control algorithm to activate or deactivate the actuators. The actuators unit is provided by a cold gas system comprising three sets of solenoid valve thrusters, mounted in the service module skin of the payload. Each set supply a constant torque that change the angular velocity around each main payload axis.

2. Mathematical Models

A mathematical model is a representation of the interested behavior of a real system through mathematical equations because it is unfeasible to represent mathematically whole behavior of a real system and/or develop the mathematical equations. So we can work with a mathematical equations set, linear or nonlinear, that it'll be called here as model. It must represent, within a known error range, the desired behavior of the real system to be controlled.

Here the mathematical models will be represented by a linear system in continuous time state space with time invariant parameters as shown at Eq. 1:

$$\begin{aligned} \boldsymbol{x}'(t) &= \boldsymbol{A}\,\boldsymbol{x}(t)+\boldsymbol{B}\,\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\,\boldsymbol{x}(t) \end{aligned} \qquad (1)$$

where $\boldsymbol{A} \in \Re^{n \times n}$ is the dynamic matrix; $\boldsymbol{B} \in \Re^{n \times p}$ is the control matrix; $\boldsymbol{C} \in \Re^{p \times r}$ is the output matrix; $\boldsymbol{x}(t) \in \Re^{n}$

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is the state vector; $\boldsymbol{u(t)} \in \Re^r$ is the control vector; and $\boldsymbol{y(t)} \in \Re^p$ is the output vector.

To facilitate the equations development, it will be omitted the terms that indicate function of t and bold letters will identify matrices and vectors.

2.1. Mathematical Model of the Payload

Payload mathematical model will represent the payload dynamic interested behavior and here it'll be called as plant. According to [1], the plant is obtained considering the following simplifications: the payload is considered as a rigid body, symmetric around body X axis, the payload will be controlled during the ballistic trajectory phase when the atmosphere influence can be neglected and it'll be designed only the control for roll, so any other angular velocity, except around body X axis, will be considered null. Base on these simplifications the plant can be represented by Eq. 2:

$$[p'] = [0]\, p + \left[\frac{l_x}{I_{xx}}\right] 2F_p \tag{2}$$

where p is the plant state, that is, the roll angular velocity ("roll-rate"), in rad/s, l_x is the arm of the actuator force and I_{xx} is the inertial momentum around body X axis and F_p is each actuator force, in N, perpendicular to the body X axis. The actuator force is multiplied by 2 because there are two actuators for roll control at opposite side of payload skin to avoiding plant translation.

2.2. Actuator Mathematical Model

Each actuator mathematical model is nonlinear compound of a linear second order system designed for rise time (t_r) of 50ms, at 10% of error below the steady state force, and damping ratio of 1.0, [2], and a nonlinear ON/OFF control. The actuator when turned on will supply a force of 2N, for its input m_a equal a 1 (ON). Here it won't be considered the hysteresis and the electromechanical delay of actuators subsystems that may affect the actuator response. Based on these simplifications the actuator model can be represented by the equation Eq. 3:

$$\begin{bmatrix} x'_{a1} \\ x'_{a2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6006.25 & -155 \end{bmatrix} \begin{bmatrix} x_{a1} \\ x_{a2} \end{bmatrix} + \begin{bmatrix} 0 \\ 6006.25 \end{bmatrix} m_a$$
$$F_p = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{a1} \\ x_{a2} \end{bmatrix} \tag{3}$$

where $\boldsymbol{x_a}$ is the actuator state vector and m_a is the magnitude input control: value 1 (ON), corresponding to output of 2N, or value 0 (OFF).

The actuator response to a pulse command of 0.3s is showed at Fig. 2. The input signal is represented by a dash line and the actuator force response is represented by the solid line.

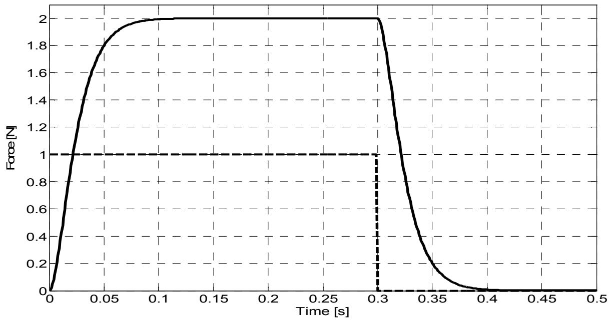

Figure 2 – Actuator response to a unitary pulse.

2.3. Model Parameters Uncertainties

If uncertainties about the model or unmeasured inputs to the process are structured [3], that is, it is known how they enter at the system dynamics; this information can be incorporated into the model. In the linear case, if model uncertainties are supposed to be structured the model can be represented by Eq. 4:

$$\boldsymbol{x}' = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x} + (\boldsymbol{B} + \Delta\boldsymbol{B})[\,\boldsymbol{u} + \Delta\boldsymbol{u}\,]$$
$$\boldsymbol{y}(t) = (\boldsymbol{C} + \Delta\boldsymbol{C})\boldsymbol{x} + \boldsymbol{s_a} \tag{4}$$

where $\Delta\boldsymbol{A}$, $\Delta\boldsymbol{B}$ and $\Delta\boldsymbol{C}$ denotes plant parameters uncertainties, $\Delta\boldsymbol{u}$ is the input parameter uncertainty and $\boldsymbol{s_a}$ is the sensor additive noise measurement.

3. RCS Design

There are many methods to design a control law for this kind of RCS. To improve the performance of the RCS to reduce the payload angular velocity as close as possible to zero it is necessary to work also with the actuator transient response. The difficult in this case is how to obtain a system that can determine the pulse width during the steady state or the transient response of the actuator.

There is a class of control law where the controlling signal can take on only two or three values. A typical example of this type of control for an ideal RCS is a relay control with dead zone as showed at Fig. 3.

The actuators pair can work at three conditions: ONActP (Increase Angular Velocity), OFFAct or ONActN (Decrease Angular Velocity). The control input is a function of plant state. For ideal case, the control law can be the curve showed at Fig. 3 that may be represented by Eq. 5.

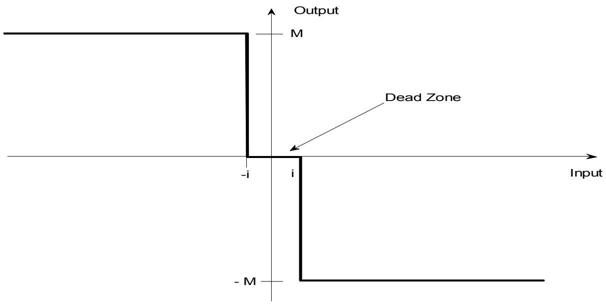

Figure 3 – Ideal rate control system.

$$m_a = \begin{cases} 0 & ,|p| <= i \\ -M\,sgn(p) & ,|p| > i \end{cases} \qquad (5)$$

where sgn( ) is the sign function and M is the scalar magnitude of the actuator force.

3.1. State Plane Method

State Plane Method is a graphical method presented by Poincaré to get the solution of simultaneous differential equations of first order [2, 4]. If we take a plane with the coordinates x and x', the equation x' can be linear or nonlinear functions of the state variable x. In this method each one of the plant state corresponds to a point at this plane. The curve formed by these points is called trajectory.

For microgravity experiments the desired plant state is the origin of the state plane. If the ideal switching function of Eq. 5 is applied to the plant equation, the state plane is divided into two halves and it will be linear in each, [2, 4], and it is said to be piecewise linear. The desired trajectories can be determined by combining linear trajectory on each side of the state plane that moves the plant state to the state plane origin. Considering an ideal case where the switching function of Fig. 3 is applied to the plant, the plant equation may be represented by Eq. 6:

$$p' = 2m_a \frac{l_x}{I_{xx}} \qquad (6)$$

The state plane for such a system for various constant values of m_a is showed at Fig. 4.

3.2. Control Law Design

Considering now the case where the actuator presents a transient response as showed at Fig. 2, we can get, based in the time-optimal control, [2, 4], the optimal trajectory that moves the plant state variables to the state plane origin when the actuator is turned off, as showed at Fig. 5.

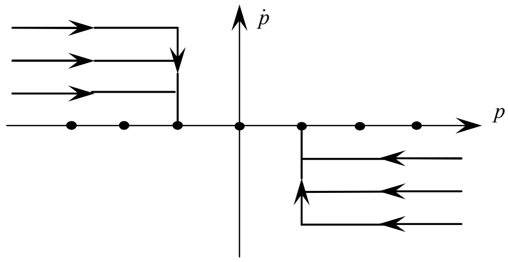

Figure 4 – State plane for a relay control system.

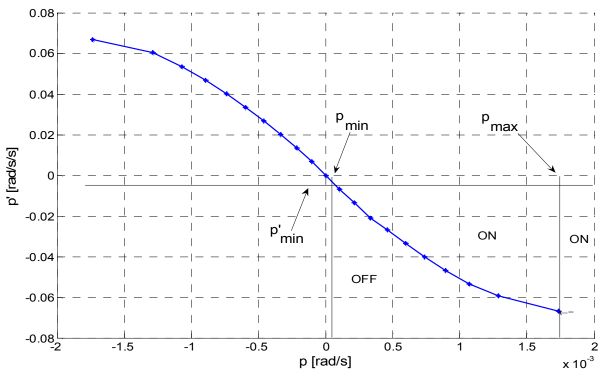

Figure 5 – Optimal trajectories that moves the plant state variable to the state plane origin.

These trajectories will be used as a switch function at the control algorithm and they'll be used instead of the dead zone of the ideal case. The points of these trajectories, used by the control algorithm, are listed at Tab. 1. This switch function will be used as a boundary to turn on or to turn off the actuators. As sensors measures are corrupted by noise, the control system is a digital process and the actuator presents a transient when it is turn off, so, it is necessary to test plant variable state to some additional boundaries. To make easy the description of this decision function it'll be considered only the control for positive angular velocity. One boundary is related to values greater than the maximum positive p value of the switch function p_{max}, where the negative actuator must be turned on. Another is related to values smaller than the minimum positive angular velocity, p_{min}, that is related to the angular velocity sensor sensibility, where the negative actuator must be turned off. And finally is related to the values smaller than the minimum negative angular acceleration, $-p'_{min}$, that may have some significant influence to decrease the payload angular velocity where the positive actuator can't be turned on, even if the angular velocity is negative. Based on these considerations and at Fig. 5 it can be designed a Decision Function that doesn't use the variable time to obtain the pulse width to get the trajectory that moves the plant state variables to the origin of the state plane.

Table 1- Points of switch function

Angular Velocity [rad/s]	Angular Acceleration [rad/s^2]
0.0017332	-0.066950
0.0012876	-0.060324
0.0010709	-0.053593
0.0008949	-0.046941
0.0007381	-0.040244
0.0005945	-0.033535
0.0004593	-0.026745
0.0003357	-0.020145
0.0002163	-0.013403
0.0001039	-0.006682
0.0	0.0
-0.0001039	0.006682
-0.0002163	0.013403
-0.0003357	0.020145
-0.0004593	0.026745
-0.0005945	0.033535
-0.0007381	0.040244
-0.0008949	0.046941
-0.0010709	0.053593
-0.0012876	0.060324
-0.0017332	0.06695

The Decision Function is an algorithm that will compare the sensors measurement of the state variable against the Switch Function (*sf)* and boundaries described for Fig. 5. The Control Algorithm needs to detect when a trajectory pass cross the switching curve or the boundaries to turn on or turn off the actuators.

The state variables used for the decision function are the measures obtained by the angular velocity and acceleration sensors: *p* and *p'*. The outputs of the Decision Function to be implemented at the Control Algorithm are the actuators commands: ONActP (turn on the positive actuator to increase angular velocity), OFFActP (turn off the positive actuator), ONActN (turn on negative actuator to decrease angular velocity) and OFFActN (turn off the negative actuator). The Decision Function algorithm is listed below:

```
If p>=p_max then ONActN
else if p>p_min or p'<-p'_min then
    if p>p_min then
        if p'>sf(p) then ONActN
        else OFFActN
    else OFFActN
else if |p|<p_min and |p'|<p'_min then
    OFFActN
    OFFActP
else if p<-p_max then ONActP
else if p<-p_min or p'>p'_min then ONActP
    if p<-p_min then
        if p'<sf(p) then ONActP
        else OFFActP
    else OFFActP
```

4. RCS PERFORMANCE

To show the RCS performance it is considered the actuator response showed at Fig. 2 and the plant parameters, for Eq. 2, given by Eq. 7:

$$l_x = 0.5\text{m} \qquad I_{xx} = 30\text{kgm}^2 \tag{7}$$

The sensor noise is modeled as a color noise with Gaussian distribution, given by Eq. 8, where its amplitude is maintained constant during each integration step.

$$f(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{8}$$

where μ = 0rad/s is the noise mean and $\sigma = 2e^{-5}$rad/s is the noise standard deviation.

The RCS performance was verified through simulation and for this it was desired to get the payload angular velocity reduced to $\pm 5e^{-4}$rad/s, that for l_x=0.5m, it represents a centripetal acceleration within the range of $\pm 1.25e^{-7}$m/s^2.

The simulations were executed by Simulink version 6.3 of MatLab version R14 SP3 with integration step of 1ms. The RCS performance was tested for the cases below.

4.1. Case I

First case is considering plant initial angular velocity of 0.05rad/s. The RCS performance for this plant initial angular velocity is showed at Figs. 6-7. At Fig. 6 the upper graph shows time response for plant angular velocity behavior, middle graph is a magnify of upper graph to see the behavior within the desired range and lower graph is the plant acceleration that it's proportional to the actuator control force to reduce the angular velocity.

Fig. 7 is another form to represent the controlled plant state behavior for the RCS designed. Solid line represents the plant state vector trajectory and dashed line represents the desired optimal trajectory.

4.2. Case II

This case shows the performance of the RCS when it is necessary to control the plant at the transient response of the actuator. The RCS performance for initial angular velocity of 0.001rad/s is showed at Figs. 8-9. At Fig. 8, the upper graph is the control signal, where positive values represent the commando to turn on positive actuators and negative values represent the command to turn on negative actuators. At Fig. 9, there is the plant trajectory to the desired state.

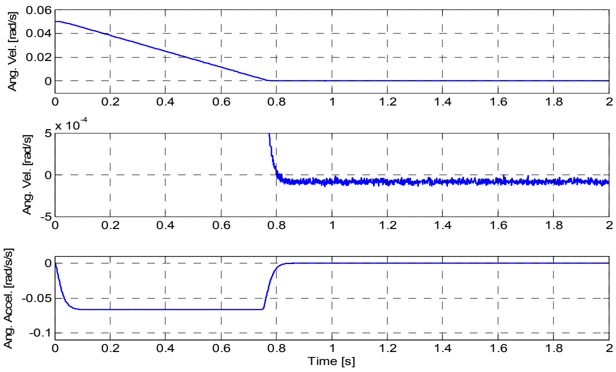

Figure 6 – Controlled plant behavior for initial angular velocity of 0.05rad/s.

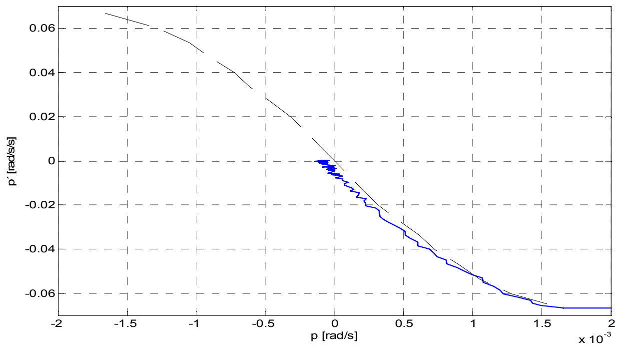

Figure 7 – Plant state vector at state plane for initial condition of 0.05rad/s.

4.3. Case III

As the RCS is designed for time-optimum trajectory, [2, 4], then it is necessary to check its robustness for parameters uncertainties during the actuator transient response. The RCS performance for a mass uncertainty of -20%, that represents an inertial momentum of I_{xx}=24, and initial angular velocity of 0.001rad/s is showed at Fig. 10.

4.4. Case IV

RCS performance for a mass uncertainty of +20%, that represents an inertial momentum I_{xx}=36, and initial angular velocity of 0.001rad/s is showed at Fig. 11.

4.5. Case V

RCS performance for actuator transitory time uncertainty of +30% (t_r = 65ms) and initial angular velocity of 0.001rad/s are showed at Figs. 12-13. At Fig. 12 there are the controlled plant time response graphs and at Fig. 13 it is showed the respective plant state trajectory.

4.6. Case VI

RCS performance for actuator transitory time uncertainty of -30% (t_r = 35ms) and initial angular velocity of 0.001rad/s is showed at Fig. 14.

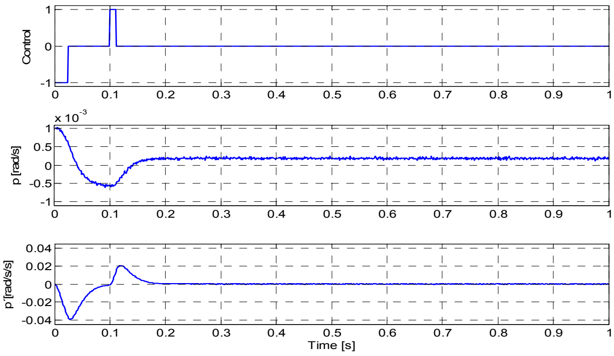

Figure 8 - Controlled plant behavior for an initial angular velocity of 0.001rad/s.

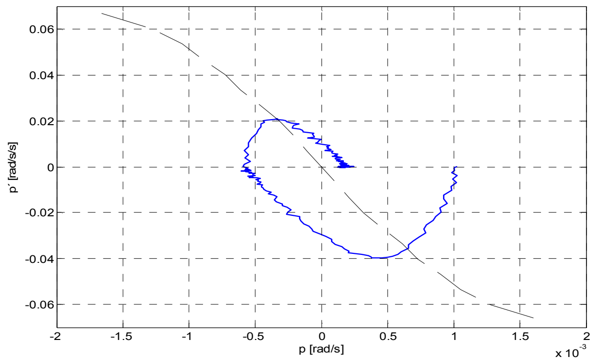

Figure 9 – Plant state vector at state plane for an initial angular velocity of 0.001rad/s.

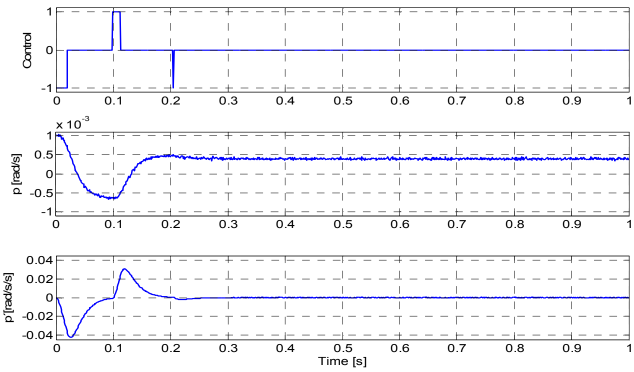

Figure 10 – Controlled plant behavior for mass uncertainty of -20% and initial angular velocity of 0.001rad/s:

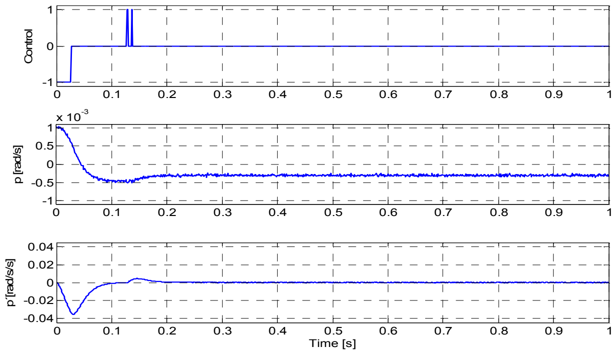

Figure 11 – Controlled plant behavior for mass uncertainty of +20% and initial angular velocity of 0.001rad/s.

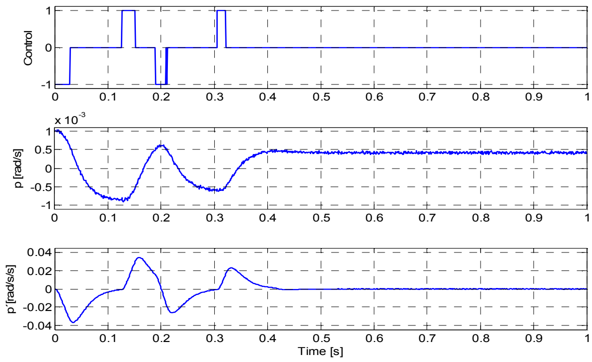

Figure 12 – Controlled plant behavior for actuator transitory time uncertainty of +30% and initial angular velocity of 0.001rad/s.

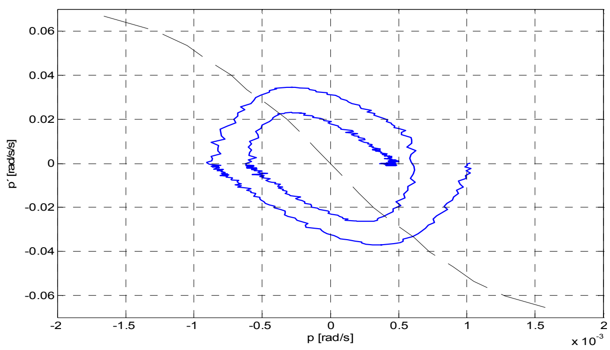

Figure 13 – Plant state vector at state plane for an initial angular velocity of 0.001rad/s.

5. CONCLUSIONS

The whole payload dynamic is very difficult or unfeasible to do the mathematical development to design a RCS. Therefore, it's used some mathematical simplifications to obtain a mathematical model, called plant, to represent the interested payload behavior. Doing so, there are parameters uncertainties at plant when compared to the real payload dynamic. These parameters uncertainties can affect the RCS performance.

A RCS designed through the state plane method showed that it is robust to plant mass and actuator force rise time uncertainties. Its performance was tested successfully at simulations with the followings parameters uncertainties: mass uncertainty range of +20% and -20% and actuator rise time range of +30% and -30%. At simulations it was also considered noise at sensors measures.

A RCS design is not complete, there are some more problems to be solved, so, the next steps are: obtain the payload angular acceleration or equivalent information to avoid using angular acceleration sensor, compensate the unknown electromechanical delay to activate and deactivate the actuators and do real time correction of the switch function during the control phase.

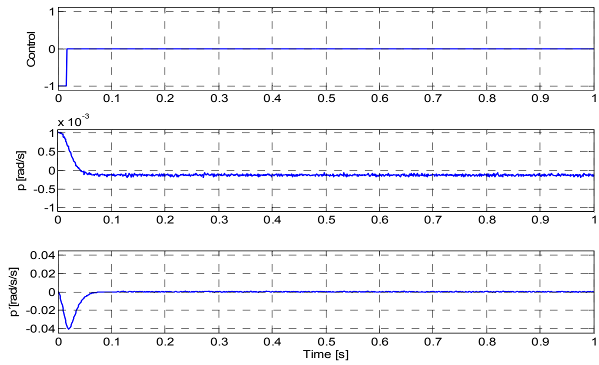

Figure 14 – Controlled plant behavior for actuator rise time uncertainty of -30% and initial angular velocity of 0.001rad/s.

6. Sample References

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